Dynamic relaxation processes in compressible multiphase flows

Application to evaporation phenomena

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Some industrial applications where evaporation is crucial

Fuel injectors at high pressure

Cryogenic injection device in space launcher

BLEVE phenomena (Security): rapid depressurization of liquefied gas + combustion
Typical physical processes we are interested in

- Liquid flow from a high pressure chamber
  - Valve/diaphragm
  - Low pressure chamber

- « Flashing » (rapid depressurization) generated by the initial pressure ratio
- Liquid/vapor mixture (drops, bubbles, pockets, …)
- Cavitation created by geometric singularities (strong rarefaction waves)
- Drops evaporation and jet explosion

Two-velocities flow (inter-penetration)

Interface problems

How can be solved the multiphase flow when the topology strongly varies?

A way is to consider relaxation effects
Plan

- Description of the non-equilibrium multiphase flow model
- Presentation of the relaxation effects
- Some multidimensional numerical results
Non-equilibrium multiphase compressible flow model
Multiphase compressible flow model

- Initially proposed by (Baer & Nunziato, IJMF, 1986) for two phases
- Each phase has its own set of equations and associated variables (velocity, pressure temperature, entropy, …):

\[
\frac{\partial \alpha_k}{\partial t} + \mathbf{U}_k \cdot \nabla \alpha = 0 \\
\frac{\partial (\alpha \rho_k)}{\partial t} + \nabla \cdot (\alpha \rho_k \mathbf{u}_k) = 0 \\
\frac{\partial (\alpha \rho u_k)}{\partial t} + \nabla \cdot \left( \alpha (\rho \mathbf{u} \cdot \mathbf{u} + P I) \right)_k = P I \cdot \nabla \alpha_k \\
\frac{\partial (\alpha \rho E_k)}{\partial t} + \nabla \cdot (\alpha (\rho E + P) \mathbf{u})_k = P I \cdot \mathbf{U}_k \cdot \nabla \alpha_k
\]

- Equation of state: \( e_k(P_k, \rho_k) \)

Discrete Equations Method: Godunov’s concept applied to multiphase cells

- **Godunov’s method** = averaging procedure by solving a single Riemann problem at each cells interface.

- **DEM** = averaging procedure by solving Riemann problems between pure fluids at each cells interface.

Fixed control volume = volume filled with the fluid

Fixed control volume = sum of the sub-volumes filled with the fluids

Each phase has a time-dependent sub-volume (volume fraction)
Major contributions of DEM (1)

⇒ Discrete equations traducing the time evolution of phases and mixture variables are obtained = Numerical scheme

⇒ When dealing with a two-phases bubbly flow, a continuous model has been obtained:

\[
\frac{\partial \alpha_1}{\partial t} + u_1 \frac{\partial \alpha_1}{\partial x} = \mu (P_1 - P_2)
\]

\[
\frac{\partial (\alpha \rho u)_1}{\partial t} + \frac{\partial (\alpha (\rho u^2 + P))}{\partial x} = P_1 \frac{\partial \alpha_1}{\partial x} + \lambda (u_2 - u_1)
\]

\[
\frac{\partial (\alpha \rho E)_1}{\partial t} + \frac{\partial (\alpha (\rho E + P)u)_1}{\partial x} = P_1 u_1 \frac{\partial \alpha_1}{\partial x} - \mu P_1 (P_1 - P_2) + \lambda u_1 (u_2 - u_1)
\]

All interface variables are determined!

\[
\mu = \frac{A_1}{Z_1 + Z_2}
\]

\[
\lambda = \frac{A_1 Z_1 Z_2}{Z_1 + Z_2}
\]

⇒ \(\mu\) and \(\lambda\) express the rates at which pressure and velocity equilibrium are reached respectively.
Major contributions of DEM (2)

- This non-equilibrium model is able to treat simultaneously:
  - mixtures with several velocities, temperatures,…
  - material interfaces (contact)
  - permeable interfaces (interfaces separating a cloud of drops and a gas for example,…)

![Image of shock wave propagation and points over time](image)
Evaporation effects

- This model is completed by heat and mass transfer source terms (evaporation, condensation) at infinite rates (relaxations).

- Relaxation processes (at infinite rate) allow the obtention of solutions in limit cases corresponding to reduced models.

- They are particularly relevant when:
  - the interfacial area and the flow topology are unknown,
  - the Direct Numerical Simulation of interface problems is considered.
Two-phase compressible flow model + relaxation terms

Hyperbolic

\[
\frac{\partial \alpha_1}{\partial t} + \bar{u}_1 \cdot \nabla \alpha_1 = 0
\]

\[
\frac{\partial (\alpha \rho \bar{u})}{\partial t} + \nabla \left( \alpha (\rho \bar{u} + P \bar{I}) \right) = P_1 \nabla \alpha_1
\]

\[
\frac{\partial (\alpha \rho E)}{\partial t} + \nabla \left( \alpha (\rho E + P) \bar{u} \right) = P_1 \bar{u}_1 \cdot \nabla \alpha_1
\]

\[
\frac{\partial (\alpha \rho \bar{u})}{\partial t} + \nabla \left( \alpha (\rho \bar{u} + P \bar{I}) \right) = -P_1 \nabla \alpha_1
\]

\[
\frac{\partial (\alpha \rho E)}{\partial t} + \nabla \left( \alpha (\rho E + P) \bar{u} \right) = -P_1 \bar{u}_1 \cdot \nabla \alpha_1
\]

Pressure relaxation

\[
\mu (P_1 - P_2)
\]

Velocity relaxation

\[
\lambda (\bar{u}_2 - \bar{u}_1)
\]

Heat transfer

\[
H_T (T_2 - T_1)
\]

Mass transfer

\[
\frac{\dot{m}}{\bar{\rho}_1} = - \dot{m}
\]

\[
- \dot{m} \tilde{\alpha}_1
\]

\[
- \dot{m} \tilde{H}_1
\]

\[
+ \dot{m}
\]

\[
+ \dot{m} \tilde{\alpha}_1
\]

\[
+ \dot{m} \tilde{H}_1
\]
Relaxation coefficients

- $\mu$ and $\lambda$ express the rate at which \textit{pressure} and \textit{velocity} equilibria are reached respectively:
  \[
  \mu(P_1 - P_2) \quad \lambda(\bar{u}_2 - \bar{u}_1)
  \]

  ➔ Mechanical interactions (drags, compressibility ratio) between phases

- $H_T$ expresses the rate at which \textit{temperature} equilibrium is reached:
  \[
  H_T(T_2 - T_1) \quad ➔ \text{Heat exchange between phases}
  \]

- $\nu$ expresses the rate at which \textit{Gibbs free energy} equilibrium is reached:
  \[
  \dot{m} = \bar{\rho}_1 \nu(g_1 - g_2) \quad ➔ \text{Mass transfer (evaporation/condensation process)}
  \]

- $\mu$, $H_T$, $\nu \rightarrow \infty$ ➔ Instantaneous exchanges: relaxation effects
Relaxation effects at infinite rates
Hierarchy of compressible flow models: reduced models

- **Homogeneous Euler model**
  - Thermal relaxation (temperature)
  - $H_T \to \infty$
  - Thermodynamic equilibrium flows
  - $(u,p,T,g)$

- **Multi-species Euler mixture model**
  - Chemical relaxation (Gibbs free energy)
  - $\nu \to \infty$
  - Thermal equilibrium flows
  - $(u,p,T,g_1,g_2)$

- **Single-velocity flow model**
  - Mechanical relaxation (velocity and pressure)
  - $\lambda, \mu \to \infty$
  - Interface problems
  - $(u,p,T_1,T_2,g_1,g_2)$

- **Multi-velocity flow model**
  - $\infty \to \mu, \nu, \lambda$
  - Non-equilibrium flows
  - $(u_1,u_2,p_1,p_2,T_1,T_2,g_1,g_2)$
Velocity relaxation procedure : $\lambda \rightarrow \infty$

\[
\begin{align*}
\frac{\partial \alpha_1}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho)_1}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho \bar{u})_1}{\partial t} &= \lambda (\bar{u}_2 - \bar{u}_1) \\
\frac{\partial (\alpha \rho E)_1}{\partial t} &= \lambda \bar{u}_1 (\bar{u}_2 - \bar{u}_1) \\
\frac{\partial (\alpha \rho)_2}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho \bar{u})_2}{\partial t} &= -\lambda (\bar{u}_2 - \bar{u}_1) \\
\frac{\partial (\alpha \rho E)_2}{\partial t} &= -\lambda \bar{u}_1 (\bar{u}_2 - \bar{u}_1)
\end{align*}
\]

Momentum and energy conservation

\[
\begin{align*}
\alpha_1 &= \text{cst} \\
Y_1 &= \frac{(\alpha \rho)_1}{\rho} = \text{cst} \\
Y_2 &= \frac{(\alpha \rho)_2}{\rho} = \text{cst} \\
\bar{u}^* &= Y_1 \bar{u}_1 + Y_2 \bar{u}_2 \\
e^*_1 &= e_1 + \frac{1}{2} (\bar{u}^* - \bar{u}_1) \cdot (\bar{u}^* - \bar{u}_1) \\
e^*_2 &= e_2 + \frac{1}{2} (\bar{u}^* - \bar{u}_2) \cdot (\bar{u}^* - \bar{u}_2)
\end{align*}
\]

EOS of both phases are not used explicitly
Pressure relaxation procedure: $\mu \to \infty$

\[
\begin{align*}
\frac{\partial \alpha_1}{\partial t} &= \mu (P_1 - P_2) \\
\frac{\partial (\alpha \rho)_{1}}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho \bar{\rho})_{1}}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho E)_{1}}{\partial t} &= -\mu \bar{P}_1 (P_1 - P_2) \\
\frac{\partial (\alpha \rho)_{2}}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho \bar{\rho})_{2}}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho E)_{2}}{\partial t} &= \mu \bar{P}_1 (P_1 - P_2)
\end{align*}
\]

Mass and energy conservation

\[
\begin{align*}
Y_1 &= \frac{(\alpha \rho)_1}{\rho} = \text{cst} \\
Y_2 &= \frac{(\alpha \rho)_2}{\rho} = \text{cst} \\
\bar{u}_1 &= \text{cst} \\
\bar{u}_2 &= \text{cst}
\end{align*}
\]

Use of EOS

\[
\begin{align*}
e_1^* &= e_1 - p^* \left( \frac{1}{1} - \frac{1}{\rho_1^*} \right) \\
e_2^* &= e_2 - p^* \left( \frac{1}{\rho_2^*} - \frac{1}{1} \right)
\end{align*}
\]

Use of EOS

\[
\begin{align*}
\rho_1^*(p^*) &= e_1^* (p^*, \rho_1^*) \\
\rho_2^*(p^*) &= e_2^* (p^*, \rho_2^*)
\end{align*}
\]

Function of the final relaxed pressure only

\[
\frac{1}{\rho} = \frac{Y_1}{\rho_1^*(p^*)} + \frac{Y_2}{\rho_2^*(p^*)}
\]

When considering ideal gas or ‘Stiffened Gas’ EOS an analytical relation is available for the pressure.
Single-velocity flow model: Interface problems

Asymptotic analysis of the full non-equilibrium model

\[ \lambda, \mu \rightarrow \infty \]

Mechanical equilibrium

\[ \begin{aligned} \ddot{u}_1 = \ddot{u}_2 = \ddot{u} \\ p_1 = p_2 = p \end{aligned} \]

\[
\frac{\partial \alpha_1}{\partial t} + \vec{u} \cdot \nabla \alpha_1 = \frac{(\rho_2 c_2^2 - \rho_1 c_1^2)}{\left( \frac{\rho_1 c_1^2}{\alpha_1} + \frac{\rho_2 c_2^2}{\alpha_2} \right)} \nabla \cdot \vec{u}
\]

\[
\frac{\partial (\alpha \rho)_1}{\partial t} + \nabla \cdot ((\alpha \rho)_1 \vec{u}) = 0
\]

\[
\frac{\partial (\alpha \rho)_2}{\partial t} + \nabla \cdot ((\alpha \rho)_2 \vec{u}) = 0
\]

\[
\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \left( \rho \vec{u} \vec{u} + p \vec{I} \right) = 0
\]

\[
\frac{\partial (\rho E)}{\partial t} + \nabla \left( (\rho E + p) \vec{u} \right) = 0
\]

Mixture variables:

\[ \rho = (\alpha \rho)_1 + (\alpha \rho)_2 \]

\[ \rho E = (\alpha \rho E)_1 + (\alpha \rho E)_2 \]

Wood sound speed:

\[ \frac{1}{\rho c_{\text{wood}}^2} = \frac{\alpha_1}{\rho_1 c_1^2} + \frac{\alpha_2}{\rho_2 c_2^2} \]

Each phase has its own temperature and Gibbs free energy

(Kapila & al., 2001; Saurel & al., 2008)
Pressure and temperature relaxation procedure: \( \mu, H_T \rightarrow \infty \)

\[
\begin{align*}
\frac{\partial \alpha_1}{\partial t} &= \mu (P_1 - P_2) \\
\frac{\partial (\alpha \rho)_1}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho \mu)_1}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho E)_1}{\partial t} &= -\mu \bar{P}_1 (P_1 - P_2) + H_T (T_2 - T_1) \\
\frac{\partial (\alpha \rho)_2}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho \mu)_2}{\partial t} &= 0 \\
\frac{\partial (\alpha \rho E)_2}{\partial t} &= \mu \bar{P}_1 (P_1 - P_2) - H_T (T_2 - T_1)
\end{align*}
\]

Mass and energy conservation

\[
\begin{align*}
Y_1 &= \frac{(\alpha \rho)_1}{\rho} = \text{cst} \\
Y_2 &= \frac{(\alpha \rho)_2}{\rho} = \text{cst} \\
\bar{u}_1 &= \text{cst} \\
\bar{u}_2 &= \text{cst} \\
\frac{1}{\rho} &= \frac{Y_1}{\rho_1^*} + \frac{Y_2}{\rho_2^*} = \text{cte} \\
e &= Y_1 e_1^* + Y_2 e_2^* = \text{cte}
\end{align*}
\]

Use of EOS

\[
\begin{align*}
\rho_k^* (p^*, T^*) \\
e_k^* (p^*, T^*)
\end{align*}
\]

When considering ideal gas or ‘Stiffened Gas’ EOS analytical relations are available for pressure and temperature
Two-species Euler model

Asymptotic analysis of the full non-equilibrium model

\[ \lambda, \mu, H_T \to \infty \quad \text{Mechanical and thermal equilibrium} \]

\[
\begin{aligned}
\frac{\partial (\alpha \rho)}{\partial t} + \nabla \cdot ((\alpha \rho) \vec{u}) &= 0 \\
\frac{\partial (\alpha \rho)}{\partial t} + \nabla \cdot ((\alpha \rho) \vec{u}) &= 0 \\
\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u} + p \vec{I}) &= 0 \\
\frac{\partial \rho E}{\partial t} + \nabla \cdot ((\rho E + p) \vec{u}) &= 0 \\
\frac{\partial \rho Y_1}{\partial t} + \nabla \cdot (\rho Y_1 \vec{u}) &= 0 \\
\frac{\partial \rho Y_2}{\partial t} + \nabla \cdot (\rho Y_2 \vec{u}) &= 0 \\
\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u} + p \vec{I}) &= 0 \\
\frac{\partial \rho E}{\partial t} + \nabla \cdot ((\rho E + p) \vec{u}) &= 0 \\
\end{aligned}
\]

Mixture variables:

\[
\begin{aligned}
\frac{1}{\rho} &= \frac{Y_1}{\rho_1} + \frac{Y_2}{\rho_2} \\
e &= Y_1 e_1 + Y_2 e_2 \\
E &= e + 1/2 \vec{u} \cdot \vec{u} \\
\end{aligned}
\]

Each phase has its own Gibbs free energy
Pressure, temperature and Gibbs free energy relaxation procedure: \( \mu, H_T, \nu \to \infty \)

\[
\begin{align*}
\frac{\partial \alpha_1}{\partial t} &= \mu (P_1 - P_2) - \frac{\dot{m}}{\rho_1}, \\
\frac{\partial (\alpha \rho)_1}{\partial t} &= -\dot{m}, \\
\frac{\partial (\alpha \rho \tilde{u})_1}{\partial t} &= -\dot{m} \tilde{u}_i, \\
\frac{\partial (\alpha \rho E)_1}{\partial t} &= -\mu \tilde{P}_1 (P_1 - P_2) + H_T (T_2 - T_1) - \dot{m} \tilde{H}_1, \\
\frac{\partial \alpha_2}{\partial t} &= \dot{m}, \\
\frac{\partial (\alpha \rho \tilde{u})_2}{\partial t} &= \dot{m} \tilde{u}_i, \\
\frac{\partial (\alpha \rho E)_2}{\partial t} &= \mu \tilde{P}_1 (P_1 - P_2) - H_T (T_2 - T_1) + \dot{m} \tilde{H}_1.
\end{align*}
\]

\[
\dot{m} = \tilde{\rho}_1 \nu (g_1 - g_2)
\]

Mass and energy conservation

Use of EOS

\[
\begin{align*}
\rho_k^* (p^*, T^*) \\
e_k^* (p^*, T^*) \\
+ \text{Gibbs free energy equality}
\end{align*}
\]

\[
T^* = T_{\text{sat}} (p^*)
\]

Latent heat of vaporization

\[
L_v (p^*)
\]

\[
\begin{align*}
Y_1^* &= \frac{1}{\rho_2^* (p^*)} - \frac{1}{\rho_1 (p^*)}, \\
Y_1^* &= h_2^* (p^*) - \left( e + \frac{p^*}{\rho} \right), \\
Y_1^* &= \frac{1}{\rho_2^* (p^*)} - \frac{1}{\rho_1^* (p^*)}.
\end{align*}
\]
Homogeneous Euler model

Asymptotic analysis of the full non-equilibrium model

$\lambda, \mu, H_T, \nu \to \infty$  \quad Mechanical and thermodynamic equilibrium

$$\begin{cases}
\frac{\partial \rho}{\partial t} + \vec{V} \cdot (\rho \vec{u}) = 0 \\
\frac{\partial \rho \vec{u}}{\partial t} + \vec{V} \cdot (\rho \vec{u} \vec{u} + p \vec{I}) = 0 \\
\frac{\partial \rho E}{\partial t} + \vec{V} \cdot ((\rho E + p) \vec{u}) = 0
\end{cases}$$

Closure relations :

$$e = Y_1 e_1 + Y_2 e_2 \quad E = e + 1/2 \vec{u} \cdot \vec{u}$$

$$T = T_{sat}(p)$$

Equilibrium sound speed :

$$\frac{1}{\rho c_{eq}^2} = \frac{1}{\rho c_{wood}^2} + \rho T \left[ \frac{Y_1}{C_{p,1}} \left( \frac{ds_1}{dp} \right)^2 + \frac{Y_2}{C_{p,2}} \left( \frac{ds_2}{dp} \right)^2 \right]$$

$\rho c_{eq}^2 < \rho c_{wood}^2$
For an arbitrary number of phases...
**Multiphase compressible flow model**

<table>
<thead>
<tr>
<th>Hyperbolic</th>
<th>Pressure relaxation</th>
<th>Velocity relaxation</th>
<th>Temperature relaxation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial \alpha_k}{\partial t} + \bar{u}_l \cdot \nabla \alpha_k = )</td>
<td>[ \sum_{l=1,N} \mu_{kl} (P_k - P_l) ]</td>
<td>[ \sum_{l=1,N} \lambda_{kl} (\bar{u}_l - \bar{u}_k) ]</td>
<td>[ \sum_{l=1,N} H_{T,kl} (T_1 - T_k) ]</td>
</tr>
<tr>
<td>( \frac{\partial (\alpha \rho)_k}{\partial t} + \nabla \cdot (\alpha \rho \bar{u}_k) = )</td>
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<td>( \frac{\partial (\alpha \rho \bar{u}_k)}{\partial t} + \nabla \cdot (\alpha (\rho \bar{u}_k + P \bar{I})) = P_1 \nabla \cdot \bar{u}_k )</td>
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</tr>
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</table>
Pressure, temperature relaxation and Gibbs free energy relaxation (liquid/vapor): $\mu_{kl} \rightarrow \infty$, $H_{T,kl} \rightarrow \infty$, $\nu \rightarrow \infty$

**Phase 1 (liquid):**

- **Pressure Relaxation**
  \[
  \frac{\partial \alpha_1}{\partial t} = \sum_{l=1,N} \mu_{ll}(P_1 - P_l) - \frac{\dot{m}}{\bar{\rho}_l}
  \]

- **Temperature Relaxation**
  \[
  \frac{\partial (\alpha \rho)}{\partial t} = -\dot{m}, \quad \frac{\partial (\alpha \rho \bar{u})}{\partial t} = -\dot{m} \tilde{H}_l
  \]

- **Gibbs Free Energy Relaxation**
  \[
  \frac{\partial (\alpha \rho E)_1}{\partial t} = -\bar{P}_l \sum_{l=1,N} \mu_{ll}(P_1 - P_l) + \sum_{l=1,N} H_{T,1l}(T_1 - T_l) - \dot{m} \tilde{H}_l
  \]

\[
\dot{m} = \bar{P}_l \nu (g_1 - g_2)
\]

**Phase 2 (vapor):**

- **Pressure Relaxation**
  \[
  \frac{\partial \alpha_2}{\partial t} = \sum_{l=1,N} \mu_{2l}(P_2 - P_l) + \frac{\dot{m}}{\bar{\rho}_l}
  \]

- **Temperature Relaxation**
  \[
  \frac{\partial (\alpha \rho)}{\partial t} = \dot{m}, \quad \frac{\partial (\alpha \rho \bar{u})}{\partial t} = \dot{m} \tilde{H}_l
  \]

- **Gibbs Free Energy Relaxation**
  \[
  \frac{\partial (\alpha \rho E)_2}{\partial t} = -\bar{P}_l \sum_{l=1,N} \mu_{2l}(P_2 - P_l) + \sum_{l=1,N} H_{T,2l}(T_1 - T_2) + \dot{m} \tilde{H}_l
  \]

**Phase k (inert):** $k \geq 3$

- **Pressure Relaxation**
  \[
  \frac{\partial \alpha_k}{\partial t} = \sum_{l=1,N} \mu_{kl}(P_k - P_l)
  \]

- **Temperature Relaxation**
  \[
  \frac{\partial (\alpha \rho)}{\partial t} = 0, \quad \frac{\partial (\alpha \rho \bar{u})}{\partial t} = 0
  \]

- **Gibbs Free Energy Relaxation**
  \[
  \frac{\partial (\alpha \rho E)_k}{\partial t} = -\bar{P}_l \sum_{l=1,N} \mu_{kl}(P_k - P_l) + \sum_{l=1,N} H_{T,kl}(T_1 - T_k)
  \]

**Relaxation:**
- Pressure
- Temperature
- Gibbs free energy
Pressure, temperature and Gibbs free energy (liquid/vapor) relaxation for an arbitrary number of phases

\[
\begin{align*}
Y_k^* &= Y_k^0 = \text{cst} \quad (k = 3, N) \\
Y_1^* + Y_2^* + \sum_{k=3,N} Y_k^0 &= 1 \\
\frac{1}{\rho} &= \frac{Y_1^*}{\rho_1^*} + \frac{Y_2^*}{\rho_2^*} + \sum_{k=3,N} \frac{Y_k^0}{\rho_k^*} \\
e &= Y_1^* e_1^* + Y_2^* e_2^* + \sum_{k=3,N} Y_k^0 e_k^*
\end{align*}
\]

Use of EOS
\[
\begin{align*}
\rho_k^*(p^*, T^*) \\
e_k^*(p^*, T^*)
\end{align*}
\]

+ Gibbs free energy equality
\[
T^* = T_{\text{sat}}(p^*)
\]

Modification of the final thermodynamic state between the liquid and its vapor

\[
Y_1^* = \frac{(Y_1^0 + Y_2^0) h_2^*(p^*) - (e + \frac{p^*}{\rho}) + \sum_{k=3,N} Y_k^0 h_k^*(p^*)}{h_2^*(p^*) - h_1^*(p^*)}
\]

\[
Y_1^* = \frac{(Y_1^0 + Y_2^0)}{\rho_2^*(p^*)} \frac{1}{\rho} + \sum_{k=3,N} \frac{Y_k^0}{\rho_k^*(p^*)} - \frac{1}{\rho_2^*(p^*)} - \frac{1}{\rho_1^*(p^*)}
\]
One-dimensional examples
Shock tube (water, steam, air)

\[ P = 10 \text{Bar} \quad T = T_{\text{sat}}(P) \approx 467 \text{ K} \]

\[ P = 1 \text{Bar} \quad T = T_{\text{sat}}(P) \approx 373 \text{ K} \]

Cloud of droplets in a gas (air/steam) mixture

\[ \alpha_{\text{Liq}} = 5 \times 10^{-4} \]

\[ \alpha_{\text{Air}} = 0.2 \]
Mechanical equilibrium
\( \lambda, \mu \to \infty \)

Mechanical/thermal equilibrium
\( \lambda, \mu, H_T \to \infty \)

Mechanical/thermodynamical equilibrium
\( \lambda, \mu, H_T, \nu \to \infty \)
Another possibility: heat exchanges between inert phases and liquid/vapor are absent

Phase 1 (liquid):

\[ \frac{\partial a_1}{\partial t} = \sum_{l=1,N} \mu_{1l}(P_1 - P_l) - \frac{\dot{m}}{\bar{\rho}_l} \]

\[ \frac{\partial (\alpha \rho)_1}{\partial t} = -\dot{m} \]

\[ \frac{\partial (\alpha \rho \bar{\nu})_1}{\partial t} = -\dot{m} \bar{\nu}_1 \]

\[ \frac{\partial (\alpha \rho E)_1}{\partial t} = -\bar{P}_1 \sum_{l=1,N} \mu_{1l}(P_l - P_1) + \sum_{l=1,N} H_{T,1l}(T_l - T_1) - \dot{m} \bar{H}_1 \]

\[ \dot{m} = \bar{\rho}_1 \nu(g_1 - g_2) \]

Phase k (inert):

\[ \frac{\partial a_k}{\partial t} = \sum_{l=1,N} \mu_{kl}(P_k - P_l) \]

Relaxation:
- pressure
- temperature

Phase 2 (vapor):

\[ \frac{\partial a_2}{\partial t} = \sum_{l=1,N} \mu_{2l}(P_2 - P_1) + \frac{\dot{m}}{\bar{\rho}_l} \]

\[ \frac{\partial (\alpha \rho)_2}{\partial t} = \dot{m} \]

\[ \frac{\partial (\alpha \rho \bar{\nu})_2}{\partial t} = \dot{m} \bar{\nu}_1 \]

\[ \frac{\partial (\alpha \rho E)_2}{\partial t} = -\bar{P}_1 \sum_{l=1,N} \mu_{2l}(P_l - P_1) + \sum_{l=1,N} H_{T,2l}(T_l - T_1) + \dot{m} \bar{H}_1 \]

Relaxation:
- pressure
- temperature

Phase 3 (inert):

\[ \frac{\partial a_3}{\partial t} = \sum_{l=1,N} \mu_{3l}(P_3 - P_l) \]

Relaxation:
- pressure
- temperature

Phase N (inert):

\[ \frac{\partial a_N}{\partial t} = \sum_{l=1,N} \mu_{Nl}(P_N - P_l) \]

Relaxation:
- pressure
- temperature

\[ \dot{m}_i = \frac{\Delta V_i}{\rho_i} \]

Relaxation:
- pressure
- temperature

Gibbs free energy:

\[ g(g_1 - g_2) = \sum_{i=1}^3 a_i \frac{\rho_i}{\Delta V_i} \]

\[ g(g_1 - g_2) = \sum_{i=1}^3 a_i \frac{\rho_i}{\Delta V_i} \]

\[ g(g_1 - g_2) = \sum_{i=1}^3 a_i \frac{\rho_i}{\Delta V_i} \]

\[ g(g_1 - g_2) = \sum_{i=1}^3 a_i \frac{\rho_i}{\Delta V_i} \]
Mechanical/thermodynamical equilibrium \( \lambda, \mu, H_T, v \to \infty \)

Thermodynamical equilibrium between liquid and vapor only
Multi-dimensional examples
Cryogenic liquid and inert gas injections

Tank filled with cryogenic liquid

\[ p = 20 \text{bar} \]

Inert gas injection

\[ p = 1 \text{Bar} \]

\(~50000\) tetrahedrons
Pressure and temperature equilibrium flow
Pressure, temperature and Gibbs free energy equilibrium flow
Vapor volume fraction evolution
Filling of cavity under gravity effect

Connection to atmosphere

Cold water

Air

P = 1 Bar

Walls

Hot steam

P = 1 Bar

Gravity
Liquid volume fraction evolution: pressure relaxation
Pressure and temperature relaxation
Pressure, temperature and Gibbs free energy relaxation
Perspectives

- Exact steady solutions through converging-diverging nozzles should be investigated under mechanical, thermal and Gibbs free energy equilibria for an arbitrary number of phases

  ➔ Reference solutions

- The knowledge of the interfacial area at each point of the flow must be strongly improved and particularly the change of the multiphase flow topology.

Additional equations are thus required (number of particles per unit volume, geometrical equations,...)

  ➔ Big challenge !!!
Thanks for your attention...