Computing water-hammer flows with a two-fluid model

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1 Introduction

2 The two-fluid model & water hammer

3 Computation of the two-fluid model
   - Evolution step
   - Relaxation step

4 Verification of the two-fluid model

5 Conclusions and perspectives
**Water-hammer transient flow in piping systems of a nuclear power plant**

**Water-hammer** *(hydraulic shock)*:
sudden change of direction or velocity → large changes in pressure  
**Risk**: damage to pipes, supports, and valve; power plant availability

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**Figure**: Water-hammer ‘classical’

**Figure**: Are line Civaux (2011)

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**Objective**: Water-hammer simulation  
(SITAR Project)

**Modeling water-hammer flows** *(Assessing/comparing the different models)*  
**Computing the model with fluid-structure interaction (FSI)** *(Europlexus)*
Modeling water-hammer flows

Complex phenomenon: column separation/cavitation...

Phenomenon characteristics:

- compressible two-phase flows (water/steam)
- unsteady flows
- shock waves
- fast transient

→ two-phase flow modeling
→ hyperbolic system
→ unique jump conditions
→ explicit scheme

Different two-phase flow models:

- two-fluid approach:
  - the two-fluid model (7 equations)
  - the two-fluid single-pressure model (6 equations)

- homogeneous approach:
  - the five-equation models (5 equations)
  - homogeneous relaxation model (4 equations)
  - homogeneous equilibrium model (3 equations)
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Governing equations of the two-fluid model

7 PDE
+ closure \((P_I, V_I)\) + EOS(equation of state): \(\varepsilon_\varphi(\rho_\varphi, p_\varphi) + \) source terms

\[
\begin{align*}
\partial_t \alpha_v & \quad + V_I \partial_x \alpha_v = \delta_v \\
\partial_t (\alpha_\varphi \rho_\varphi) & \quad + \partial_x (\alpha_\varphi \rho_\varphi u_\varphi) = \Gamma_\varphi, \ \varphi = l, v \\
\partial_t (\alpha_\varphi \rho_\varphi u_\varphi) & \quad + \partial_x (\alpha_\varphi \rho_\varphi u_\varphi^2 + \alpha_\varphi p_\varphi) - P_I \partial_x \alpha_\varphi = \Gamma_\varphi U_{int} + D_\varphi \\
\partial_t (\alpha_\varphi \rho_\varphi e_\varphi) & \quad + \partial_x [(\alpha_\varphi \rho_\varphi e_\varphi + \alpha_\varphi p_\varphi) u_\varphi] + P_I \partial_t \alpha_\varphi = \Gamma_\varphi H_{int} + D_\varphi U_{int} + Q_\varphi
\end{align*}
\]

with \(e_\varphi = \varepsilon_\varphi(\rho_\varphi, p_\varphi) + u_\varphi^2/2\), \(U_{int} = (u_l + u_v)/2\) and \(H_{int} = u_l * u_v/2\),

\[
\begin{align*}
\delta_v & \quad \text{mass transfer} & \delta_v & \quad \text{pressure relaxation} \\
\Gamma_\varphi & \quad \text{momentum transfer} & \Gamma_\varphi & \quad \text{chemical potential relaxation} \\
D_\varphi & \quad \text{velocity relaxation} & D_\varphi & \quad \text{energy transfer} \\
Q_\varphi & \quad \text{temperature relaxation}
\end{align*}
\]
Choice of closure laws for water-hammer flows

- Interfacial pressure/velocity:
  \[ P_I = p_l, \quad V_I = u_v \] (Baer-Nunziato)

- EOS:
  - Liquid: Stiffened Gas (SG)
    \[ \varepsilon_l(\rho_l, p_l) = \frac{p_l + \gamma_l(p_l)_{\infty}}{\rho_l(\gamma_l - 1)} \]
  - Vapor: Perfect Gas (GP)
    \[ \varepsilon_v(\rho_v, p_v) = \frac{p_v}{\rho_v(\gamma_v - 1)} \]

- Source terms (Entropy-consistent interfacial closure):
  - Pressure relaxation:
    \[ \delta_v = (\tau_p)^{-1} \frac{\alpha_l\alpha_v}{|P_l| + |P_v|}(P_v - P_l), \quad \sum_{\varphi = l, v} \delta_{\varphi} = 0 \]
  - Velocity relaxation:
    \[ D_v = (\tau_u)^{-1} \frac{m_l m_v}{m_l + m_v}(u_l - u_v), \quad \sum_{\varphi = l, v} D_{\varphi} = 0, \]
  with \( \tau_k, \ k = p, u \), time scales for pressure/velocity relaxations,
  \( m_{\varphi} = \alpha_{\varphi} \rho_{\varphi}, \ \varphi = l, v \), partial masses.

- Remark:
  - Liquid/vapor: \( P_I = p_l, \ (p_l)_{\infty} \gg (p_v)_{\infty} \)
  - Gas/particles: \( P_I = p_g, \ (p_g)_{\infty} \ll (p_p)_{\infty} \), (GHHN, M2AN 2010)[1]
Main properties of the two-fluid model

- **Property 1 Hyperbolicity and structure of waves**
  The system (1) is hyperbolic unless $|u_l - u_v| = c_l$. It admits 7 real eigenvalues:

  $$
  \begin{align*}
  \lambda_{1,2} &= u_v, \\
  \lambda_3 &= u_v - c_v, \\
  \lambda_4 &= u_v + c_v, \\
  \lambda_5 &= u_l, \\
  \lambda_6 &= u_l - c_l, \\
  \lambda_7 &= u_l + c_l
  \end{align*}
  $$

  Fields associated with $\lambda_{1,2,5}$ are linearly degenerate (LD). Other fields are genuinely nonlinear (GNL).

- **Property 2 Jump conditions**
  Unique jump conditions hold within each field associated with $\lambda_k$. $\alpha_l$ is uniform apart from the field associated with $\lambda_{1,2} = u_v$. Jump conditions in other fields correspond to single phase jump relations.

- **Property 3 Entropy inequality**
  Define the entropy $\eta(W) = m_l s_l + m_v s_v$ and the entropy flux $f_\eta(W) = m_l s_l u_l + m_v s_v u_v$; then smooth solutions $W$ of (1) are such that:

  $$
  0 \leq \partial_t(\eta(W)) + \partial_x(f_\eta(W))
  $$

  with the constraint $c_\varphi^2 \partial_{\rho_\varphi} (s_\varphi) + \partial_{\rho_\varphi} (s_\varphi) = 0$ for entropies $s_\varphi$. 
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A fractional step method complying with the entropy inequality:

At time interval $[t^n, t^n + \Delta t]$, given initial values $W^n$

- **Evolution step**: $W^n \xrightarrow{\text{convective effects}} \tilde{W}$

\[
\begin{aligned}
\partial_t \alpha_v + V_I \partial_x \alpha_v &= 0 \\
\partial_t (\alpha \rho \varphi) + \partial_x (\alpha \rho \varphi u \varphi) &= 0 \\
\partial_t (\alpha \rho \varphi u \varphi) + \partial_x (\alpha \rho \varphi u^2 + \alpha \rho \varphi) - P_I \partial_x \alpha \varphi &= 0 \\
\partial_t (\alpha \rho \varphi e \varphi) + \partial_x [(\alpha \rho \varphi e \varphi + \alpha \rho \varphi) u \varphi] + P_I \partial_t \alpha \varphi &= 0
\end{aligned}
\] (2)

- **Relaxation step**: $\tilde{W} \xrightarrow{\text{source terms}} W^{n+1}$

\[
\begin{aligned}
\partial_t \alpha_l &= \delta_l \\
\partial_t (\alpha_l \rho_l) &= 0 \\
\partial_t (\alpha_l \rho_l u_l) &= D_l \\
\partial_t (\alpha_l \rho_l e_l) + P_I \partial_t (\alpha_l) &= D_l U_{int} \\
\partial_t (\alpha_v \rho_v + \alpha_l \rho_l) &= 0 \\
\partial_t (\alpha_v \rho_v u_v + \alpha_l \rho_l u_l) &= 0 \\
\partial_t (\alpha \rho \varphi e \varphi + \alpha l \rho l e l) &= 0
\end{aligned}
\] (3)
Evolution step: an extension of Rusanov scheme

Homogeneous system:
\[ \partial_t W + \partial_x (f(W)) + h(W) \partial_x \alpha_v = 0 \] (4)

Extension of Rusanov scheme (SR1):

\[ h_i(W_i^{n+1} - W_i^n) + \Delta t^n (F_{i+1/2}^n - F_{i-1/2}^n) + \Delta t^n \left( (\alpha_1^n)_i^{1/2} - (\alpha_1^n)_{i-1/2}^n \right) \beta^n_i = 0 \] (5)

- Flux:
\[ F_{i+1/2}^n = \frac{1}{2} \left( f(W_i^n) + f(W_{i+1}^n) - r_{i+1/2}(W_{i+1}^n - W_i^n) \right) \] (6)

- Non-conservative terms:
\[ (\alpha_l^n)_{i+1/2}^n = \frac{1}{2} \left( (\alpha_l)_i^n + (\alpha_l)_{i+1}^n \right), \beta^n_i = h(W_i^n) \] (7)

Property 4
The scheme (5) preserves positive values of partial masses \( m_\varphi \) and void fractions \( \alpha_\varphi \), if
\[ \Delta t^n \left( r_{i+1/2} + r_{i-1/2} \right) \leq 2CFL h_i, \quad CFL \in [0, 1[ \]
The second-order Rusanov scheme (**SR1-ORDER2**)

**Time scheme** : second-order Runge-Kutta method

**Space discretisation** : minmod reconstruction \( (Y = (\alpha_v, u_v, p_v, s_v, u_l, p_l, s_l)^t) \)

If \( (\Phi_{i+1}^{n} - \Phi_{i}^{n})(\Phi_{i}^{n} - \Phi_{i-1}^{n}) > 0 \),

\[
(\Phi_{i}^{n})^+ = \Phi_{i}^{n} + \delta_{i}^{n} \frac{h_i}{2}, \quad (\Phi_{i}^{n})^- = \Phi_{i}^{n} - \delta_{i}^{n} \frac{h_i}{2},
\]

\[
\delta_{i}^{n} = \text{sign}(\Phi_{i+1}^{n} - \Phi_{i}^{n}) \min\left\{ \frac{2}{h_{i+1}+h_{i}}, 2 \left| \frac{\Phi_{i+1}^{n} - \Phi_{i}^{n}}{h_{i+1}+h_{i}} \right| \right\}
\]  \hspace{1cm} (8)

Else, \( (\Phi_{i}^{n})^+ = (\Phi_{i}^{n})^- = \Phi_{i}^{n} \)

**Flux** :

\[
F_{i+1/2}^{n} = \frac{1}{2} \left\{ f(W((Y_{i}^{n})^+)) + f(W((Y_{i+1}^{n})^-)) - r_{i+1/2} \left\{ W((Y_{i+1}^{n})^-) - W((Y_{i}^{n})^+) \right\} \right\}
\]  \hspace{1cm} (9)

**Non-conservative terms** :

\[
h(W)\partial_x \alpha_v \rightarrow \frac{\left( \alpha_1 \right)_{i+\frac{1}{2}}^n - \left( \alpha_1 \right)_{i-\frac{1}{2}}^n}{h_i} \beta_i^n
\]  \hspace{1cm} (10)
Verification of the schemes: test cases

- Test case 1 (TT, JCP, 2010; SWK, JCP, 2006) [6, 4]
  2 contact discontinuities, 1 rarefaction, 3 shocks
  EOS: GP(vapor), SG (liquid)

- Test case 2 (constructed),
  A moving void fraction wave
  EOS: GP(vapor), SG (liquid)

\[ \text{Figure:} \] Structure of fields of case 2

- Test case 3 (constructed),
  A moving void fraction wave
  EOS: GP(vapor), SG (liquid)
  Large variation of \( \alpha_v \): 0.05 \( \rightarrow \) 0.95, equilibrium of other variables
Verification of the first/second-order Rusanov schemes: test case 1

**Figure:** SR1, SR1-ORDER2; with 200 & 500 cells, $CFL = 0.49$, $t = 0.15$. 
Verification of the first/second order Rusanov scheme: test case 1

Rate of convergence observed:

SR1 : \( \frac{1}{2} \), \( \text{SR1-ORDER2} : \frac{2}{3} \)

**Figure:** L-1 norm of the error for variables \( \alpha_v, \rho_\varphi, u_\varphi, p_\varphi, \varphi = l, v \) traced in logarithmic scale, \( \text{SR1, SR1-ORDER2, CFL} = 0.49, t = 0.15 \)
Verification of the first/second order Rusanov scheme: test case 2

**Figure**: SR1, SR1-ORDER2; 200 & 500 cells, $CFL = 0.49$, $t = 0.25$
Verification of the first/second order Rusanov scheme : test case 2

Rate of convergence observed:
SR1 : $\frac{1}{2}$, \quad SR1-ORDER2 : $\frac{2}{3}$

**Figure:** SR1, SR1-ORDER2, $CFL = 0.49, t = 0.25
Evolution step: a fractional step method

\[
\begin{aligned}
\partial_t \alpha_v + u_v \partial_x \alpha_v &= 0 \\
\partial_t (\alpha_\varphi \rho_\varphi) &= 0, \quad \varphi = l, v \\
\partial_t (\alpha_\varphi \rho_\varphi u_\varphi) &= 0, \quad \varphi = l, v \\
\partial_t (\alpha_\varphi E_\varphi) - p_l u_v \partial_x \alpha_\varphi &= 0, \quad \varphi = l, v
\end{aligned}
\]

(11)

\[
\begin{aligned}
\partial_t \alpha_v &= 0 \\
\partial_t (\alpha_\varphi \rho_\varphi) + \partial_x (\alpha_\varphi \rho_\varphi u_\varphi) &= 0, \quad \varphi = l, v \\
\partial_t (\alpha_\varphi \rho_\varphi u_\varphi) + \partial_x (\alpha_\varphi \rho_\varphi u_\varphi^2) + \partial_x (\alpha_\varphi p_\varphi) - p_l \partial_x \alpha_\varphi &= 0, \quad \varphi = l, v \\
\partial_t (\alpha_\varphi E_\varphi) + \partial_x [(\alpha_\varphi E_\varphi + \alpha_\varphi p_\varphi) u_\varphi] &= 0, \quad \varphi = l, v
\end{aligned}
\]

(12)

Property
System (11) is hyperbolic. It admits seven real eigenvalues. All fields are LD.

\[
\lambda_1 = u_v, \lambda_{2-7} = 0
\]

System (12) is hyperbolic unless: \(|u_l| = c_l\), or \(|u_v| = c_v\). It admits seven real eigenvalues. The 1, 2, 5-fields are LD and other fields are GNL.

\[
\begin{aligned}
\lambda_1 &= 0, \\
\lambda_2 &= u_v, \lambda_3 = u_v - c_v, \lambda_4 = u_v + c_v, \\
\lambda_5 &= u_l, \lambda_6 = u_l - c_l, \lambda_7 = u_l + c_l.
\end{aligned}
\]
Evolution step: a fractional step method

First step: $W^n \rightarrow W^*$

\[
\begin{align*}
\partial_t \alpha_v + u_v \partial_x \alpha_v &= 0 \\
\partial_t (\alpha_\varphi \rho_\varphi) &= 0, \; \varphi = l, v \\
\partial_t (\alpha_\varphi \rho_\varphi u_\varphi) &= 0, \; \varphi = l, v \\
\partial_t (\alpha_\varphi E_\varphi) - p_l u_v \partial_x \alpha_\varphi &= 0, \; \varphi = l, v
\end{align*}
\]

(13)

$\alpha_v$: Rusanov flux formulation

\[
\begin{align*}
(\alpha_v E_v + \alpha_l E_l)_i^* &= (\alpha_v E_v + \alpha_l E_l)_i^n \\
(s_l)_i^* &= (s_l)_i^n
\end{align*}
\]

(14)

Second step: $W^* \rightarrow W^{n+1}$: generic Rusanov formulation

Property

The scheme preserves positive values of partial masses $m_\varphi$ and void fractions $\alpha_\varphi$ if the CFL like condition holds on. The scheme preserves the mean total energy.
Verification of the fractional step method : test case 1

Rate of convergence observed :
PFRAC3 : \( \frac{1}{2} \),

\[\text{Figure: } \text{SR1, PFRAC3, } CFL = 0.49, t = 0.15.\]
Verification of the fractional step method: test case 2

Rate of convergence observed:
PFRAC3: \( \frac{1}{2} \),

\[
\begin{align*}
\text{Figure: } & \text{SR1, PFRAC3 } CFL = 0.49, \ t = 0.25 .
\end{align*}
\]
Verification of the fractional step method: test case 3

Rate of convergence observed:
PFRAC3 : $\frac{1}{2}$,

**Figure:** PFRAC3, CFL = 0.49, $t = 0.25$. 
Relaxation step

Computation of the two-fluid model

\[ \tilde{W} \xrightarrow{\text{velocity relaxation}} W^* \xrightarrow{\text{pressure relaxation}} W_i^{n+1} \]  

(15)

Computing the ordinary differential equations:

Velocity relaxation: drag terms

\[ \begin{align*}
    \partial_t \alpha_l &= 0 \\
    \partial_t (\alpha_l \rho_l) &= 0 \\
    \partial_t (\alpha_l \rho_l u_l) &= D_l \\
    \partial_t (\alpha_l \rho_l e_l) &= D_l U_{int} \\
    \partial_t (\alpha_v \rho_v + \alpha_l \rho_l) &= 0 \\
    \partial_t (\alpha_v \rho_v u_v + \alpha_l \rho_l u_l) &= 0 \\
    \partial_t (\alpha_v \rho_v e_v + \alpha_l \rho_l e_l) &= 0 
\end{align*} \]

(16)

Proposed in [2]  
(HH, CaF, 2012)

Pressure relaxation:

\[ \begin{align*}
    \partial_t \alpha_v &= \delta_v \\
    \partial_t (\alpha_v \rho_v) &= 0 \\
    \partial_t (\alpha_v \rho_v u_v) &= 0 \\
    \partial_t (\alpha_v \rho_v e_v) + p_l \partial_t \alpha_v &= 0, \\
    \partial_t (\alpha_l \rho_l) &= 0 \\
    \partial_t (\alpha_l \rho_l u_l) &= 0 \\
    \partial_t (\alpha_l \rho_l e_l) + p_l \partial_t \alpha_l &= 0, 
\end{align*} \]

(17)

Proposed in [1]  
(GHHN, M2AN, 2010)  
+ Extension to new EOS
Pressure relaxation: extension to EOS GP(vapor)+SG(liquid)

The ordinary differential equation (17) is:

\[
\begin{align*}
\partial_t \alpha_v &= \delta_v \\
\partial_t (m_v e_v) + p_l \partial_t \alpha_v &= 0 \\
\partial_t (m_l e_l) - p_l \partial_t \alpha_v &= 0 \\
\partial_t (\alpha \rho \varphi) &= 0 \\
\partial_t (\alpha \rho \varphi u \varphi) &= 0
\end{align*}
\]

(18)

An implicit scheme calculates \((p_l^{n+1}, p_v^{n+1}, \alpha_v^{n+1})\), solution of

\[
\begin{align*}
\alpha_v^{n+1} - \alpha_v^* + \Delta t(\theta)^{-1} \alpha_l^{n+1} \alpha_v^{n+1} (p_l^{n+1} - p_v^{n+1}) &= 0 \\
(m_l e_l)^{n+1} - (m_l e_l)^* - P_l^{n+1} (\alpha_v^{n+1} - \alpha_v^*) &= 0 \\
(m_v e_v)^{n+1} - (m_v e_v)^* + P_l^{n+1} (\alpha_v^{n+1} - \alpha_v^*) &= 0
\end{align*}
\]

(19)

**Property 6**

Assume that the EOS of the gas phase is GP, and that of the liquid phase is SG, then the scheme (19) admits a unique relevant solution \((p_l^{n+1}, p_v^{n+1}, \alpha_v^{n+1})\) such that \(p_v > 0\), \(p_l + (p_l)_\infty > 0\) and \(\alpha_v^{n+1}\) lies in \([\alpha_M, \alpha_m] \in [0, 1]\), with

\[
\begin{align*}
\alpha_M &= \frac{\gamma_v - 1}{\gamma_v} \alpha_v^0, \\
\alpha_m &= \frac{1 + (\gamma_l - 1) \alpha_v^0}{\gamma_l}
\end{align*}
\]
Verification of the pressure relaxation scheme

Slope of curves : 0.56
Rate of convergence expected : 1
‘Exact solution’ : Matlab ODE45 approach (AbsTol : 1e-14)

**Figure**: Curves of convergence of the pressure relaxation scheme.
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**Simpson Experiment (in process)**

**Figure:** Simpson Experiment

**Figure:** Pressure at the valve of pipe

Combination of many different phenomena:

- Reflection of waves, vaporous cavitation
- Regime transition: almost single-phase flow → mixture of water/steam
- Fluid structure interaction

**Preliminary questions:** Is the two-fluid model able to

- retrieve the single-phase phenomena at the beginning of experiment?
- capture the speed of sound in the liquid/vapor mixture?
Single-phase phenomena of Simpson experiment

**Figure:** Pressure magnitude of the first shock wave: \( p = \alpha_l p_l + \alpha_v p_v \) according to different values of \( \alpha_l \)

Joukowsky formula: incompressible single-phase estimation

\[
\Delta p = \rho_l c_l \Delta u
\]  

(20)

Numeric: \( \tau_p = 10^{-10} \text{s}, \tau_u = 10^{-10} \text{s}, \)
Verification of the two-fluid model

Speed of sound in the liquid/vapor mixture

**Figure:** Numerics, Wallis, experiment for different values of $\alpha_l$

**Wallis formula:**

\[
\frac{1}{mc_{Wallis}^2} = \frac{\alpha_l}{\rho_l c_l^2} + \frac{\alpha_v}{\rho_v c_v^2}
\]  

(21)

**Experiment:** [3] (Karplus, H. B.-1958)

**Numerics:** $\tau_p = 10^{-10} \text{s}, \tau_u = 10^{-10} \text{s},$

**Remark:** There is no definition of speed of sound for the mixture for the two-fluid model, here $c_{num} = \text{dis}_{pulse}/\text{time}$
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Conclusions and perspectives

- Two-fluid model:
  - stable convection schemes + relaxation schemes \((p, u)\)
  - rather expensive (3D computation!!!)
  - good approximation of speed of sound in the mixture
  - single-phase phenomena retrieved
  - masse transfer relaxation scheme (cavitation) to be tested
  - more complete realistic calculations to be leaded

- The five-equation models

- Comparison of models + FSI
Thank you for your attention!

Any questions?

Hyperbolic relaxation models for granular flows.


J.-M. Hérard and O. Hurisse.

A fractional step method to compute a class of compressible gas-liquide flows.


H. B. Karplus.

The velocity of sound in a liquide containing gas bubbles.


D. Schwendeman, C. Wahle, and A. Kapila.

The Riemann problem and a high-resolution Godunov method for a model of compressible two-phase flow.


A. R. Simpson.

*Large water hammer pressures due to column separation in sloping pipes (transient cavitation).*

S. Tokareva and E. Toro.

HLLC-type Riemann solver for the Baer-Nunziato equations of compressible two-phase flow.

*Journal of Computational Physics, 229:3573–3604, 2010.*